

Anomalous effect of disorder on spin fluctuations in non-centrosymmetric superconductors

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The spin susceptibility tensor $\chi_s^{ij}(T)$ of an impure superconductor (SC) with broken mirror symmetry has been evaluated and a great effect of impurity scattering has been shown. As opposed to conventional singlet superconductors, where the ordinary impurity scattering is known to have no effect on $\chi_s(T)$, the spin susceptibility of a polar symmetry superconductor with s -wave pairing can be isotropic and equal to its value in the normal state in the “dirty” limit $T_c\tau \ll 1$, while the superconductor stays in a full-gap state. The effect is bound up with spin-flip transitions which accompany the electron scattering in conductors with the band spin-orbit coupling.

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I. INTRODUCTION

It is well known that the true nature of the superconducting state appears more clearly in an external magnetic field. The major part of phenomena induced by the field, such as the Meissner effect and the vortex formation in type-II materials, is intimately connected with the orbital degrees of freedom of electrons. The importance of the interaction of the electron spins with the magnetic field begins to be increasingly recognized with the appearance of the microscopic theory of superconductivity.¹ It was found that the Zeeman energy contributes essentially to the thermodynamic balance of superconductors^{2,3} and can result in the spatially dependent order parameter.⁴ Our concern here is the spin susceptibility $\chi_s(T)$. According to the BCS theory, $\chi_s(T)$ of a conventional (centrosymmetric with the s -wave pairing) superconductor vanishes in the ground state ($T=0$). This prediction originally made for pure materials⁵ has been extended to alloys soon—the ordinary impurity scattering should not influence $\chi_s(T)$.^{6,7} So a theoretical picture emerged: all $\chi_s(T)$ at $T < T_c$ is due to quasiparticles and the condensate of singlet Cooper pairs is indifferent to the Zeeman interaction. However, results of measurements on the Knight shift showed a departure from this simple picture. While $\chi_s(T)$ of light metals such as aluminum agreed with the theory,⁸ experiments on more heavy metals, e.g., mercury⁹ and tin,¹⁰ indicated that $\chi_s(0)$ is finite for these superconductors. It was supposed¹¹ that the spin-orbit interaction could be responsible for the finite value of $\chi_s(0)$. A rigorous theory developed subsequently has confirmed the hypothesis—the account of the spin-orbit component of the amplitude of scattering on nonmagnetic impurities $u(\mathbf{p}, \mathbf{p}') = a(\mathbf{p} - \mathbf{p}') + ib(\mathbf{p} - \mathbf{p}')p_F^{-2}(\mathbf{p} \times \mathbf{p}') \cdot \boldsymbol{\sigma}$ does lead to a finite value of $\chi_s(0)$, leaving the critical temperature T_c unchanged.¹² It was thought for a some time that the paper¹² reconciles the BCS theory to the experiments and that impurities and grain boundaries are the only sources of the spin-orbit coupling at low temperatures, when phonons are frozen out. Things have drastically changed recently when superconductivity in compounds with broken central symmetry has been discovered and has become the focus of intense research for uncommon properties that could reveal such materials (for a review see, e.g., Ref. 13).

The absence of central symmetry is equivalent to the existence of the intracrystalline electric field that gives rise to a term in the one-particle Hamiltonian that (in the case of the nondegenerate conduction band) is linear in the spin vector $\boldsymbol{\sigma}$ and linear or cubic in the electron momentum \mathbf{p} . In the case of SCs of polar symmetry, which the following consideration is restricted to, this term has the form¹⁴

$$H_{\text{so}} = \alpha(\mathbf{p} \times \mathbf{c}) \cdot \boldsymbol{\sigma}, \quad (1)$$

where \mathbf{p} and $\boldsymbol{\sigma}$ are, respectively, the electron momentum and the Pauli matrices, the unit vector \mathbf{c} points along the polar axis, and units in which $\hbar = c = k_B = 1$ are used. We shall term the Hamiltonian (1) the band spin-orbit coupling (BSOC) as opposed to the impurity spin-orbit coupling (ISOC) considered in Ref. 12. Examples of bulk SCs of polar symmetry are Mo_3AlC (symmetry $P4_132$), $\text{La}_5\text{B}_2\text{C}_6$ (symmetry $P4$), Mo_3P (symmetry $I\bar{4}$)¹⁵ and the ternary silicides CeCoSi_3 , LaRhSi_3 , and LaIrSi_3 (symmetry $I4$ mm).¹⁶ The local breaking of “up-down” symmetry may also occur in quasi two-dimensional (2D) systems, e.g., near the interface of two different superconductors or a superconductor (SC) and a normal metal,¹⁷ and at the surface of insulating WO_3 when the surface is doped by Na^+ ions.¹⁸ In those cases the vector \mathbf{c} is one of two nonequivalent normals to a 2D system. Thus, the BSOC is an additional and possibly the most powerful source of the spin-orbit coupling in conductors with broken mirror symmetry.

Naturally one should expect an effect of the BSOC on spin-dependent properties of the SCs, in particular, on the susceptibility. Indeed, it results in a finite value of $\chi_s(0)$:¹⁹ calculations carried out for 2D SCs have shown that $\chi_{s\parallel}(0) = 2\chi_{s\perp}(0) = \frac{2}{3}(\frac{\alpha p_F}{\Delta})^2 \chi_n$ at $\alpha p_F \ll \Delta$ (Ref. 20) and $\chi_{s\parallel}(0) = 2\chi_{s\perp}(0) = \chi_n$ at $\alpha p_F \gg \Delta$,²¹ where $\chi_{s\parallel}$ ($\chi_{s\perp}$) is the spin susceptibility for a magnetic field parallel (perpendicular) to the polar axis \mathbf{c} , Δ is the value of the quasiparticle gap at $T=0$, and χ_n is the spin susceptibility of the metal in the normal state. Analogous results have been derived for 3D SCs afterwards.²² Measurements, however, show the temperature dependence of $\chi_s^{ij}(T)$ at $T < T_c$ to be negligible.^{23,24}

Several explanations of this behavior have been proposed. One of those attributes it to the effect of strong many-body interactions.²⁵ There is, however, a subtle pitfall in that approach which lies in the fact that the quasiparticle decay of

the Fermi surface does not allow one to prove the Hermitian property of the renormalized one-particle Hamiltonian, where the consideration of Ref. 25 is based on, and hence to ensure the unitarity of the theory. So it is unclear whether the standard framework of the Fermi liquid theory will be a useful approach in that case. The enhancement of $\chi_s(0)$ (accompanied by the dominance of p - or d -wave pairing) was also found as a consequence of the antiferromagnetic order.²⁶ As is known, an analysis of a system with two instability channels (the spin-density-wave channel responsible for the antiferromagnetic order and the Cooper channel) suggests the account of their mutual influence to be self-consistent, i.e., requires the “parquet” approach.²⁷ Yet the antiferromagnetic order is introduced in Ref. 26 phenomenologically; the effect of the spin-density wave correlations on the pairing interaction and the effect of superconducting correlations on the spin-density-wave correlations are neglected. The weak temperature dependence of $\chi(T)$ could also be a consequence of the presence of nodes in the energy gap^{28,29} which can occur if the admixture of the triplet order parameter, which is possible due to the BSOC,²⁰ is comparable in magnitude to the singlet order parameter.

It should be noted that the temperature independent χ_s is apparently not a property of a particular non-centrosymmetric SC but rather their common feature. Such a behavior was observed, for example, in tetragonal CePt₃Si (Refs. 23 and 24) which exhibits the heavy-fermion behavior and antiferromagnetic order ($T_n \approx 2$ K) beyond the superconducting transition ($T_c = 0.7$ K), but also in cubic LiPt₃Si (Ref. 30) which shows no evidence of magnetic order or strongly correlated electron effects up to the superconducting transition at $T_c \approx 4$ K.³¹ The assumption about nodes in the energy gap^{28,29} appears to be universal enough so that it could be applicable to many non-centrosymmetric SCs; however, it supposes the magnitude of the BSOC αp_F (where p_F is the Fermi momentum) to be as large as the Fermi energy ϵ_F . It should be noticed that all the theories mentioned proceed on the assumption that the SCs are pure, i.e., $\tau T_c \gg 1$, where τ is the elastic relaxation time and T_c is the critical temperature. In this paper, another one reason for the absence of a temperature dependence of $\chi_s(T)$ at $T < T_c$ is pointed out. It is disorder.

There are quite a few reasons to consider effects of disorder on SCs without center of inversion. First, scattering centers are present in most physical situations. Parity-odd SCs have complex composition, some of them are synthesized by the arc melting method so that the crystal structure of the materials does not have to be perfect. Second, the impurity scattering essentially determines not only equilibrium properties of the impure SCs but also their nonequilibrium properties when the scattering provides the main channel for the momentum relaxation. Finally, there is a special reason to include disorder into physics of parity-odd SCs—in addition to the momentum relaxation, the scattering on a scalar, *spin-independent* potential simultaneously gives rise to the spin relaxation. This reason for the spin relaxation can be most easily explained by using the semiclassical language. For an electron with the momentum \mathbf{p} , the term (1) can be considered as the Zeeman energy of the electron in a fictitious magnetic field $\mathbf{B}_f(\mathbf{p}) = \alpha(\mathbf{p} \times \mathbf{c}) / \mu_B$, where μ_B is the Bohr

magneton and the electron g factor equal to two is assumed. Hence the spin of the electron precesses about $\mathbf{B}_f(\mathbf{p})$. If, as a result of scattering, the electron goes from a state with the momentum \mathbf{p} into a state with the momentum \mathbf{p}' , its spin will appear under the action of the field $\mathbf{B}_f(\mathbf{p}')$ and will have to precess about the new direction. In this way, a stochastic process of impurity scattering induces a corresponding stochastic process of the fictitious magnetic-field reorientation leading to a stochastic disturbance of the phase of the spin precession. The randomization of the spin phase results in a finite time of “forgetting” by the electron of its initial spin orientation that reveals itself through the spin magnetization decay. The described process is the basis of the D’yakonov-Perel’ (DP) theory of spin relaxation in semiconductors without inversion center.³² When the period of the Larmor precession in the fictitious field is much longer than the collision time τ , the DP theory gives the spin-relaxation time $\tau_{s0} \approx \tau \eta^2$, where $\eta = 2\alpha p_F \tau$. So the frequent electron-impurity collisions slow down the spin-density relaxation $\tau_{s0} \sim \tau^{-1}$. A rigorous microscopic quantum theory developed later³³ has confirmed this result. The essential role of the DP process in spin-dependent phenomena in non-centrosymmetric semiconductors is now widely recognized,³⁴ the same may be expected in the case of non-centrosymmetric normal metals and superconductors. In particular, the DP process should influence the spin fluctuations of the superconducting condensate at any type of pairing and consequently affect the spin response of a parity-odd superconductor to a magnetic field. In fact, it is shown below that in the “dirty” limit (when $\tau T_c \ll 1$, i.e., the mean electron free path $v_F \tau$ is much smaller than the coherence length $\xi_0 = v_F / 2\pi T_c$), the impurity scattering drives the spin susceptibility of such a superconductor to that of the normal state, while, just as in conventional SCs, the scattering has no effect on T_c and the superconductor stays in a full-gap state.

It should be noticed that recent papers^{35,36} also discuss effects of impurities upon non-centrosymmetric SCs, but substantially differ in the treatment of processes of the electron-impurity scattering. As is known, the BSOC lifts the spin degeneracy of the conduction electrons forming the energy branches with positive and negative helicities—the projections of the spin of an electron with the momentum \mathbf{p} on the direction $\mathbf{c} \times \mathbf{p}$. Considerations of those papers were based on the assumption that the scattering does result in transitions between the branches. Explicit calculations, however, do not confirm this hypothesis. The one-particle Green’s function averaged over impurities positions is diagonal in the helicity index. However, off-diagonal components of two-particle Green’s function which determines $\chi_s(T)$ appear to be just as important as diagonal components. All possible scattering channels will be shown to contribute comparably into the susceptibility and hence should be considered on equal footing. The exclusion of some scattering channels leads to the violation of the unitarity of the scattering S matrix and is a questionable approximation. The effect of impurities on the spin susceptibility within a model similar to that used in this paper but with the neglect of the interbranch transitions was also studied in Ref. 29.

In Sec. II, we describe a simple model based on the assumption of a relatively weak spin-orbit coupling $\alpha p_F \ll \epsilon_F$.

The model is mathematically tractable and, we believe, captures much of essential physics of polar SCs. We also evaluate analytically the static uniform spin susceptibility tensor both for 2D and 3D electron systems. Section III contains a discussion of the results obtained, the relevance of the theory developed to real materials, and suggestions for further experiments. Two Appendixes are included. In Appendix A, shown is the presence of the triplet correlations in the model considered in the main body of the paper in which the pairing interaction operates only in the singlet s -wave channel. In Appendix B we outline a more general formalism, that in addition to the s -wave pairing, includes the pairing in the p -wave channel, and give an estimate for the effect of impurities on the triplet part of the order parameter.

II. MODEL AND CALCULATION

To relate to metal surface states and interfaces as well as layered crystals, we assume first that the system under consideration is two-dimensional (2D). A modification for three-dimensional (3D) systems will be given in the end of this section. The physical model we are using is based on the following premises. We assume that the spectrum of the electrons in the absence of the interparticle interaction and without reference to the broken “up-down” symmetry is isotropic $\epsilon_0(p) = p^2/2m$ and the potential of impurities placed in arbitrary distributed points \mathbf{R}_i of concentration n_{imp} is short-ranged $U(\mathbf{r}-\mathbf{R}_i) = U\delta(\mathbf{r}-\mathbf{R}_i)$ (then the elastic lifetime τ is given by $\tau^{-1} = mn_{\text{imp}}U^{237}$). So let us consider a 2D system of electrons with the Hamiltonian $H = H_0 + H_{\text{imp}} + H_{\text{pair}}^{(s)}$, where

$$H_0 = \int d^2r \{ \nabla \psi_\gamma^+(\mathbf{r}) \cdot \nabla \psi_\gamma(\mathbf{r}) / 2m + \alpha \psi_\beta^+(\mathbf{r}) (-i \nabla \times \mathbf{c}) \cdot \boldsymbol{\sigma}_{\beta\gamma} \psi_\gamma(\mathbf{r}) \}, \quad (2)$$

$$H_{\text{imp}} = \sum_i \int d^2r \psi_\beta^+(\mathbf{r}) U \delta(\mathbf{r} - \mathbf{R}_i) \psi_\beta(\mathbf{r}), \quad (3)$$

$$H_{\text{pair}} = \frac{1}{2} \int d^2r_{1234} \psi_\alpha^+(\mathbf{r}_1) \psi_\beta^+(\mathbf{r}_2) V^{\alpha\beta|\gamma\delta} \times (\mathbf{r}_1, \mathbf{r}_2 | \mathbf{r}_3, \mathbf{r}_4) \psi_\delta(\mathbf{r}_4) \psi_\gamma^+(\mathbf{r}_3). \quad (4)$$

Here, $\psi_\gamma(\mathbf{r})$ is the electron field operator and the superscript + denotes Hermitian conjugation. In this paper we shall adopt the conventional character of pairing

$$V_s^{\alpha\beta|\gamma\delta}(\mathbf{r}_1, \mathbf{r}_2 | \mathbf{r}_3, \mathbf{r}_4) = \lambda_s g_{\alpha\beta} g_{\gamma\delta}^t \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{r}_3 - \mathbf{r}_4) \delta(\mathbf{r}_1 - \mathbf{r}_3), \quad (5)$$

where the superscript t denotes transposition, and hence the singlet order-parameter matrix $\Delta_{\alpha\beta}^{(s)} = \Delta_{(s)} g_{\alpha\beta}$, where $g = i\sigma_2$. Several remarks are in order in connection with the Hamiltonian (5).

(i) The constant α of the BSOC enters the problem under study in several ways. First, the energy spectrum of H_0 consists of two branches of positive and negative helicities (the projection of a spin on the $\mathbf{p} \times \mathbf{c}$ direction) with energies

$\epsilon_{(\pm)}(p) = p^2/2m \pm \alpha p$. The two branches have different Fermi momenta $p_\pm \cong p_F(1 \mp \delta)$ and different densities of states at the Fermi level $N_\pm \cong (1 \mp \delta)N(0)$, where $p_F = (2m\epsilon_F)^{1/2}$, $N(0) = \frac{m}{2\pi}$ and $\delta = \alpha p_F/2\epsilon_F$. Thus, the parameter δ accounts for the difference between the left- and right-handed electrons. General reasoning points to small δ —the constant α has a relativistic origin, whereas the Fermi energy has the atomic scale. Accordingly, in this paper, we shall treat δ as being small so that all powers of δ in excess of the first can be ignored. Other two dimensionless parameters are $\kappa = \alpha p_F/\pi T_c$ and $\eta = 2\alpha p_F\tau$. The first of them characterizes the value of the spin-orbital coupling with respect to superconducting correlations. The second is proportional to an angle through which the spin of an electron (with the momentum \mathbf{p}) rotates about the effective magnetic field $\mathbf{B}_f(\mathbf{p})$ in time τ between two successive collisions; it controls the kinetics of spin-flip processes by impurity scattering.³² Both of them are much greater than δ : $\delta/\kappa \approx T_c/\epsilon_F \ll 1$ and $\delta/\eta \approx (\epsilon_F\tau)^{-1} \ll 1$.

(ii) The main concept arisen with the beginning of parity-odd SCs is that of the singlet-triplet mixing. One should distinguish between two aspects of the concept. The two-particle interaction includes all spherical harmonics (s, p, d, \dots). Here we shall assume that the strongest attraction takes place for electron pairs in the singlet s -wave channel (the dominant channel) and, for definiteness, that the interaction of electrons in other channels is repulsive. A conventional SC would have the pure singlet order parameter (the gap matrix) $\Delta_{\alpha\beta}^{(s)} = \Delta_{(s)} g_{\alpha\beta}$ under such an assumption. But because the BSOC (1) spoils the classification of the Cooper pairs in terms of the total spin, the order parameter of a parity-odd SC can be a mixture of the singlet, $\Delta_{\alpha\beta}^{(s)}$, and triplet, subdominant component $\Delta_{\alpha\beta}^{(t)}(\mathbf{p}) = \Delta_{(t)}(\hat{\mathbf{p}} \times \mathbf{c} \cdot \boldsymbol{\sigma})_{\alpha\gamma} g_{\gamma\beta}$.²⁰ The triplet correlations occurring in that way will be referred to as the secondary triplet correlations (STC). The triplet channel of the pairing, however, is not a unique source of the triplet correlations. The point is that the major element of the pairing theory is the off-diagonal Green function (ODGF), $F_{\alpha\beta}(\mathbf{r}, \mathbf{r}') = \langle \psi_\alpha(\mathbf{r}) \psi_\beta(\mathbf{r}') \rangle$, which manifests a spontaneous breakdown of the $U(1)$ gauge invariance, rather than the order parameter. The spin structure of the ODGF is identical to that of the gap matrix $\Delta_{\alpha\beta}$ in the case of a SC with the inversion center. By contrast, the ODGF of a non-centrosymmetric SC contains the triplet component even at the singlet order parameter (see Appendix A). A reason for these triplet correlations which will be referred to as the primary triplet correlations (PTC) is broken space parity of the normal state of the SC.

(iii) The roles of the PTC and STC are totally different in a SC with small δ . First, in the case of a pure SC it has been shown that the triplet component is much smaller than the singlet one²⁰ $\frac{\Delta_{(t)}}{\Delta_{(s)}} \sim \delta \frac{\lambda_p}{\lambda_s}$, where λ_s and λ_p are the constants the pairing interaction in s -wave and p -wave channels. An examination of the Feynman's diagrams contributing to χ_s reveals that $\Delta_{(t)}$ enters χ_s being squared. Therefore, if one drops corrections of the order of δ^2 , one may put $\Delta_{(t)} = 0$. This is equivalent to retaining only the s -wave pairing channel (5). All the mentioned results²⁰⁻²² on $\chi_s(T)$ of clean SCs was obtained in this approximation. One can show that the

same estimate for $\Delta_{(t)}/\Delta_{(s)}$ holds true also for an impure SC (see Appendix B). Moreover, in the dirty limit the relative role of $\Delta_{(t)}$ is additionally reduced by the factor τT_c . Therefore, just as for clean SCs, one can disregard $\Delta_{(t)}$ by evaluating the spin susceptibility. In this way we come to the minimal model defined by Eqs. (2)–(5). An advantage of this model is that it admits an exact analytic approach and therefore can serve as a testing area for a search for and an investigation of uncommon physical properties of superconductors with broken mirror symmetry. Many of novel effects, such as the magnetoelectric effect^{37,38} and an unusual phase of the condensate induced by an applied magnetic field^{20,39} was first found by making use of the minimal model. This model, however, is insufficient for study systems with $\delta \sim 1$ in which an effect of the STC may be substantial.^{28,29} For example, accidental nodes in the quasiparticle gap^{28,29} which require comparable values of $\Delta_{(t)}$ and $\Delta_{(s)}$ cannot be obtained within the model. Some comments on such a situation will be given in the Sec. III. Here, we shall use the model to try out classical methods of treating impure SCs (Refs. 6 and 12) to systems with the BSOC and carry the DP ideas to superconductors. Although we shall consider an equilibrium problem, methods used will be undoubtedly useful at an evaluation of some electrodynamic and kinetic consequences of broken mirror symmetry when the account of the momentum and spin relaxation is necessary.

We now turn to the evaluation of the susceptibility. The thermal Green's function (propagator) \hat{G} is a 4×4 matrix in the space of a direct product of the Nambu-space (with the basic set $\tau_0, \tau_1, \tau_2, \tau_3$) with the spin space (with the basic set $\sigma_0, \sigma_1, \sigma_2, \sigma_3$). To avoid confusion the trace in the 2×2 spin space will be denoted by Tr_2 and the trace in the 4×4 space will be denoted by Tr_4 . In the case of the pointlike impurity potential and in the Born approximation to impurity scattering, the propagator \hat{G} has the same matrix structure as that found for clean SCs.²⁰ Namely,

$$\hat{G}(i\epsilon_n, \mathbf{p}) = \begin{pmatrix} G(i\epsilon_n, \mathbf{p}) & F(i\epsilon_n, \mathbf{p}) \\ F^+(-i\epsilon_n, \mathbf{p}) & -G^t(-i\epsilon_n, -\mathbf{p}) \end{pmatrix}, \quad (6)$$

$$G_{\alpha\beta}(i\epsilon_n, \mathbf{p}) = \sum_{\nu=\pm} \Pi_{\alpha\beta}^{(\nu)}(\mathbf{p}) G_{(\nu)}(i\epsilon_n, p), \quad (7)$$

$$F_{\alpha\beta}(i\epsilon_n, \mathbf{p}) = \sum_{\nu=\pm} \Pi_{\alpha\gamma}^{(\nu)}(\mathbf{p}) F_{(\nu)}(i\epsilon_n, p) g_{\gamma\beta}, \quad (8)$$

with

$$\begin{pmatrix} G_{(\nu)}(i\epsilon_n, p) \\ F_{(\nu)}(i\epsilon_n, p) \end{pmatrix} = \frac{1}{(i\hat{\epsilon}_n)^2 - \xi_{(\nu)}^2 - \tilde{\Delta}_{(s)}^2(n)} \begin{pmatrix} i\tilde{\epsilon}_n + \xi_{(\nu)} \\ \tilde{\Delta}_{(s)} \end{pmatrix}. \quad (9)$$

Here, the definitions of the 2×2 spin-matrix Green's functions G and F are the same as in Ref. 40. The operator

$$\Pi^{(\pm)}(\mathbf{p}) = \frac{1}{2} [\sigma_0 \pm (\hat{\mathbf{p}} \times \mathbf{c}) \cdot \boldsymbol{\sigma}] \quad (10)$$

represents the projection onto states with a definite helicity, $\xi_{(\pm)}(p) = \epsilon_{(\pm)}(p) - \mu$, $\epsilon_n = \pi T(2n+1)$ (n takes on integral values), and the functions $\tilde{\epsilon}_n$ and $\tilde{\Delta}_{(s)}$ are assumed to depend

only on ϵ_n but not on \mathbf{p} . Equations for the c -number functions $\tilde{\epsilon}_n$ and $\tilde{\Delta}_{(s)}$ in terms of ϵ_n and $\Delta_{(s)}$ as well as the self-consistency equation for the order-parameter matrix have the standard form⁴¹

$$i\tilde{\epsilon}_n \sigma_{\alpha\beta}^0 = i\epsilon_n \sigma_{\alpha\beta}^0 - \frac{1}{m\tau} \sum_{\mathbf{p}} G(i\epsilon_n, \mathbf{p})_{\alpha\beta}, \quad (11)$$

$$\tilde{\Delta}_{\alpha\beta}^{(s)} = \Delta_{\alpha\beta}^{(s)} - \frac{1}{m\tau} \sum_{\mathbf{p}} F(i\epsilon_n, \mathbf{p})_{\alpha\beta}, \quad (12)$$

$$\Delta_{\alpha\beta}^{(s)} = T \sum_{\epsilon_n, \mathbf{p}} \lambda_s F(i\epsilon_n, \mathbf{p})_{\alpha\beta}. \quad (13)$$

Note that spin-matrix structure of all constituents of the theory (propagators and vertices) comes from two sources. The first one, that is present in the case of conventional SCs as well, is the spin dependence of the order-parameter matrix $\Delta_{\alpha\beta}^{(s)} = \Delta_{(s)}(T) g_{\alpha\beta}$, i.e., originates in the Pauli principle. An additional source distinctive of the mirror-odd SCs is the BSOC. It is convenient to eliminate g matrices from the basic equations of the theory and operate with objects whose spin structure is only due to the BSOC. It can be achieved by means of the transformation

$$\hat{G} = \hat{X}^{-1} \hat{G}' \hat{X}, \hat{X} = \begin{pmatrix} \sigma_0 & 0 \\ 0 & g \end{pmatrix}. \quad (14)$$

Then,

$$\hat{G}'_{\alpha\beta}(i\epsilon_n, \mathbf{p}) = \sum_{\nu=\pm} \Pi_{\alpha\beta}^{(\nu)}(\mathbf{p}) \begin{pmatrix} G_{(\nu)}(i\epsilon_n, p) & F_{(\nu)}(i\epsilon_n, p) \\ F_{(\nu)}(i\epsilon_n, p) & -G_{(\nu)}(-i\epsilon_n, p) \end{pmatrix}. \quad (15)$$

In deriving Eq. (15) the use has been made of the equality

$$g \Pi^{t(\pm)}(-\mathbf{p}) g^t = \Pi^{(\pm)}(\mathbf{p}), \quad (16)$$

which is a consequence of the easily verified identity $g \boldsymbol{\sigma}^t g^t = -\boldsymbol{\sigma}$.

It can be shown that H_{s0} has no effect upon Eq. (13) to an accuracy of $\delta^2 \ll 1$, so that $\Delta_{(s)}(T)$ is the solution of the standard BCS equation for a classic impure SC,⁶ and

$$\frac{\tilde{\Delta}_{(s)}(n)}{\Delta_{(s)}} = \frac{\tilde{\epsilon}_n}{\epsilon_n} = u(\epsilon_n) \stackrel{\text{def}}{=} 1 + (2\tau \sqrt{\epsilon_n^2 + \Delta_{(s)}^2})^{-1}. \quad (17)$$

Consequently, the impurity scattering does not reduce T_c to the same accuracy.

According to general rules of quantum statistics,⁴¹ the spin susceptibility at temperature T is given by the expression

$$\chi_{s,ij}(T) = \frac{1}{2} T \sum_{\epsilon_l, \mathbf{p}} \text{Tr}_4 \{ \hat{m}_i \hat{G}(\epsilon_l, \mathbf{p}) \hat{M}_j(i\epsilon_l) \hat{G}(\epsilon_l, \mathbf{p}) \}, \quad (18)$$

where $\hat{m}_i = \frac{1}{2} \mu_B [(\tau_0 + \tau_3) \times \sigma_i - (\tau_0 - \tau_3) \times \sigma_i']$ is the bare spin-magnetic-moment vertex,⁴⁰ μ_B is the Bohr magneton, \hat{M}_j is the dressed 4×4 vertex, and the fact was used that the vertex function \hat{M}_j depends only on ϵ_l in the case of pointlike im-

purities. The ladder-type equation for the dressed vertex is¹²

$$\hat{M}_i(i\epsilon_l) = \hat{m}_i + \frac{1}{m\tau} \sum_{\mathbf{p}} \hat{\tau}_3 \hat{G}(i\epsilon, \mathbf{p}) \hat{M}_i(i\epsilon_l) \hat{G}(i\epsilon, \mathbf{p}) \hat{\tau}_3, \quad (19)$$

were $\hat{\tau}_i = \tau_i \times \sigma_0$.

Let us consider first the susceptibility for a magnetic field parallel to the polar axes \mathbf{c} , i.e., set $i=3$ in Eq. (19). The necessary steps to solve this equation are the following:

(i) Introducing the representation $\hat{M}_3 = \mu_B \hat{X}^{-1} \hat{M}' \hat{X}$, we can rewrite the equation in the form

$$\hat{M}'(i\epsilon_l) = \tau_0 \times \sigma_3 + \frac{1}{m\tau} \sum_{\mathbf{p}} \hat{\tau}_3 \hat{G}'(i\epsilon, \mathbf{p}) \hat{M}'(i\epsilon_l) \hat{G}'(i\epsilon, \mathbf{p}) \hat{\tau}_3. \quad (20)$$

In terms of the Nambu-components

$$\hat{M}' = \begin{pmatrix} M'^{(11)} & M'^{(12)} \\ M'^{(21)} & M'^{(22)} \end{pmatrix}. \quad (21)$$

Equation (20) is the 4×4 system of equations for spin matrices $M'^{(ij)}$

$$\begin{aligned} M'_{\rho\beta}{}^{(11)} &= \sigma_{3,\rho\beta} + [P \otimes M'^{(11)} + Q \otimes M'^{(12)} \\ &\quad + Y \otimes M'^{(21)} + R \otimes M'^{(22)}]_{\rho\beta}, \\ M'_{\rho\beta}{}^{(12)} &= -[Q \otimes M'^{(11)} + V \otimes M'^{(12)} \\ &\quad + R \otimes M'^{(21)} + K \otimes M'^{(22)}]_{\rho\beta}, \\ M'_{\rho\beta}{}^{(21)} &= -[Y \otimes M'^{(11)} + R \otimes M'^{(12)} \\ &\quad + U \otimes M'^{(21)} + L \otimes M'^{(22)}]_{\rho\beta}, \\ M'_{\rho\beta}{}^{(22)} &= \sigma_{3,\rho\beta} + [R \otimes M'^{(11)} + K \otimes M'^{(12)} \\ &\quad + L \otimes M'^{(21)} + T \otimes M'^{(22)}]_{\rho\beta}. \end{aligned} \quad (22)$$

Here, the kernels P, Q, \dots are the p integrals of the bilinear tensor products of two Nambu-components of the propagator \hat{G}'

$$\begin{pmatrix} P \\ Q \\ Y \\ R \\ V \\ U \\ K \\ L \\ T \end{pmatrix} = \frac{1}{\pi N(0)} \sum_{\mathbf{p}} \begin{pmatrix} G(i\epsilon, \mathbf{p})_{\rho\kappa} G(i\epsilon, \mathbf{p})_{\alpha\beta} \\ G(i\epsilon, \mathbf{p})_{\rho\kappa} F'(i\epsilon, \mathbf{p})_{\alpha\beta} \\ F'(i\epsilon, \mathbf{p})_{\rho\kappa} G(i\epsilon, \mathbf{p})_{\alpha\beta} \\ F'(i\epsilon, \mathbf{p})_{\rho\kappa} F'(i\epsilon, \mathbf{p})_{\alpha\beta} \\ G(i\epsilon, \mathbf{p})_{\rho\kappa} (-1) G(-i\epsilon, \mathbf{p})_{\alpha\beta} \\ (-1) G(-i\epsilon, \mathbf{p})_{\rho\kappa} G(i\epsilon, \mathbf{p})_{\alpha\beta} \\ F'(i\epsilon, \mathbf{p})_{\rho\kappa} (-1) G(-i\epsilon, \mathbf{p})_{\alpha\beta} \\ (-1) G(-i\epsilon, \mathbf{p})_{\rho\kappa} F'(i\epsilon, \mathbf{p})_{\alpha\beta} \\ G(-i\epsilon, \mathbf{p})_{\rho\kappa} G(-i\epsilon, \mathbf{p})_{\alpha\beta} \end{pmatrix}, \quad (23)$$

$N(0) = m/2\pi$ is the 2D density of states, and the symbol \otimes denotes the action of a kernel on a (ij) Nambu-component of \hat{M}' according to the rule

$$(P \otimes M'^{(ij)})_{\rho\beta} = P_{\rho\kappa|\alpha\beta} M'_{\kappa\alpha}{}^{(ij)}. \quad (24)$$

The evaluation of the kernels P, Q, \dots can be carried out in a manner similar to that of the Feynman diagram evaluation in spinor electrodynamics. In the integrals over the momentum space, one should change the Cartesian coordinates for polar coordinates. The angular integration gives rise to a combination of Pauli matrices, and the remaining radial integration can be elementarily performed with the help of the theory of residues. We shall carry out the calculation in detail for the kernel R , since the evaluation of other kernels is completely analogous. By making use of Eqs. (7)–(9) and (15), we have

$$R = \sum_{\mu\nu=\pm} R_{(\mu\nu)} r_{(\mu\nu)}, \quad (25)$$

where the indices in the parentheses are helicities of the electronic states and

$$R_{(\mu\nu)} = \int \frac{d\hat{p}}{2\pi} \Pi_{\rho\kappa}^{(\mu)}(\mathbf{p}) \Pi_{\alpha\beta}^{(\nu)}(\mathbf{p}), \quad (26)$$

$$r_{(\mu\nu)} = \int \frac{d\xi(p)}{\pi} F_{(\mu)}(i\epsilon, p) F_{(\nu)}(i\epsilon, p). \quad (27)$$

Using Eq. (10), we obtain

$$R_{(\mu\nu)} = \frac{1}{4} \left[\sigma_{\rho\kappa}^0 \sigma_{\alpha\beta}^0 + \frac{1}{2} \mu\nu (\mathbf{c} \times \boldsymbol{\sigma})_{\rho\kappa} (\mathbf{c} \times \boldsymbol{\sigma})_{\alpha\beta} \right], \quad \mu, \nu = \pm 1. \quad (28)$$

Note that the right-hand sides of Eqs. (26) and (28) include the direct products of a matrix which depends on the indices $(\rho\kappa)$ and a matrix which depends on the indices $(\alpha\beta)$. It is much convenient, however, to deal with the direct products of matrices one of which depends on the indices $(\rho\beta)$ and another one on the indices $(\alpha\kappa)$. In the Feynman-diagram language, it means to split up the four spin indices into a pair of indices by means of which the kernel is connected with other part of a ladder diagram coming from the left and a pair of other indices through which the kernel is connected with a part of the ladder diagram coming from the right. An advantage of the representation obtained in such a way is that it allows one to readily reduce the spin-matrix equation for the spin-vertex function to a system of scalar equations. The desired rearrangement of the spin indices is possible owing to Fierz-like identities for the direct products of the Pauli matrices.^{42,43} Identities needed have the form

$$\begin{aligned} \sigma_{\alpha\beta}^0 \sigma_{\gamma\lambda}^0 &= \frac{1}{2} [\sigma_{\gamma\beta}^0 \sigma_{\alpha\lambda}^0 + (\mathbf{n} \cdot \boldsymbol{\sigma})_{\gamma\beta} (\mathbf{n} \cdot \boldsymbol{\sigma})_{\alpha\lambda} \\ &\quad + (\mathbf{n} \times \boldsymbol{\sigma})_{\gamma\beta}^n (\mathbf{n} \times \boldsymbol{\sigma})_{\alpha\lambda}^n], \end{aligned} \quad (29)$$

$$(\mathbf{n} \times \boldsymbol{\sigma})_{\alpha\beta}^n (\mathbf{n} \times \boldsymbol{\sigma})_{\gamma\lambda}^n = \sigma_{\gamma\beta}^0 \sigma_{\alpha\lambda}^0 - (\mathbf{n} \cdot \boldsymbol{\sigma})_{\gamma\beta} (\mathbf{n} \cdot \boldsymbol{\sigma})_{\alpha\lambda}, \quad (30)$$

where \mathbf{n} is any 3D vector. By means of these identities $R_{(\mu\nu)}$ can be transformed to the form

$$R_{(\mu\nu)} = \frac{1}{4} \begin{pmatrix} \boldsymbol{\sigma}_{\rho\beta}^0 \boldsymbol{\sigma}_{\alpha\kappa}^0 + \frac{1}{2} (\mathbf{c} \times \boldsymbol{\sigma})_{\rho\beta}^n (\mathbf{c} \times \boldsymbol{\sigma})_{\alpha\kappa}^n & (\mathbf{c} \cdot \boldsymbol{\sigma})_{\rho\beta} (\mathbf{c} \cdot \boldsymbol{\sigma})_{\alpha\kappa} + \frac{1}{2} (\mathbf{c} \times \boldsymbol{\sigma})_{\rho\beta}^n (\mathbf{c} \times \boldsymbol{\sigma})_{\alpha\kappa}^n \\ (\mathbf{c} \cdot \boldsymbol{\sigma})_{\rho\beta} (\mathbf{c} \cdot \boldsymbol{\sigma})_{\alpha\kappa} + \frac{1}{2} (\mathbf{c} \times \boldsymbol{\sigma})_{\rho\beta}^n (\mathbf{c} \times \boldsymbol{\sigma})_{\alpha\kappa}^n & \boldsymbol{\sigma}_{\rho\beta}^0 \boldsymbol{\sigma}_{\alpha\kappa}^0 + \frac{1}{2} (\mathbf{c} \times \boldsymbol{\sigma})_{\rho\beta}^n (\mathbf{c} \times \boldsymbol{\sigma})_{\alpha\kappa}^n \end{pmatrix}_{(\mu\nu)}. \quad (31)$$

Further, substituting Eq. (9) into Eq. (27), we obtain

$$r_{(\mu\nu)} = \int \frac{d\xi(p)}{\pi} \frac{|\tilde{\Delta}_{(s)}|^2}{[(i\tilde{\epsilon}_n)^2 - \xi_{(\mu)}^2 - \tilde{\Delta}_{(s)}^2][(i\tilde{\epsilon}_n)^2 - \xi_{(\nu)}^2 - \tilde{\Delta}_{(s)}^2]} = \frac{\tilde{\Delta}_{(s)}^2}{2\sqrt{\tilde{\epsilon}_n^2 + \tilde{\Delta}_{(s)}^2}} \begin{pmatrix} \frac{1-\delta}{\tilde{\epsilon}_n^2 + \tilde{\Delta}_{(s)}^2} & \frac{1}{(\alpha p_F)^2 + \tilde{\epsilon}_n^2 + \tilde{\Delta}_{(s)}^2} \\ 1 & \frac{1+\delta}{\tilde{\epsilon}_n^2 + \tilde{\Delta}_{(s)}^2} \end{pmatrix}_{(\mu\nu)}. \quad (32)$$

The substitution of Eqs. (31) and (32) into Eq. (25) finally gives

$$R = \frac{\tilde{\Delta}_{(s)}^2}{4\tilde{E}_n} w, \quad (33)$$

where $\tilde{E}_n = u(\epsilon_n)E_n = \sqrt{\tilde{\epsilon}_n^2 + \tilde{\Delta}_{(s)}^2}$, $E_n = \sqrt{\epsilon_n^2 + \Delta_{(s)}^2}$ and

$$w = \frac{\boldsymbol{\sigma}_{\rho\beta}^0 \boldsymbol{\sigma}_{\alpha\kappa}^0}{\tilde{E}_n^2} + (\mathbf{c} \times \boldsymbol{\sigma})_{\rho\beta}^n (\mathbf{c} \times \boldsymbol{\sigma})_{\alpha\kappa}^n \left(\frac{1/2}{\tilde{E}_n^2} + \frac{1/2}{(\alpha p_F)^2 + \tilde{E}_n^2} \right) + \frac{(\mathbf{c} \cdot \boldsymbol{\sigma})_{\rho\beta} (\mathbf{c} \cdot \boldsymbol{\sigma})_{\alpha\kappa}}{(\alpha p_F)^2 + \tilde{E}_n^2}. \quad (34)$$

Analogous calculations yield

$$P = T = R, \quad Q = Y = R = K = L = \frac{i\tilde{\epsilon}_n \tilde{\Delta}_{(s)}}{4\tilde{E}_n} w,$$

$$V = U = -\frac{2\tilde{\epsilon}_n^2 + \tilde{\Delta}_{(s)}^2}{4\tilde{E}_n} w. \quad (35)$$

The above calculations show two things. The first one is that the contributions to R (and also to P, Q, \dots), from processes at which a pair of quasiparticles of given helicities scatter into states with any possible helicities, are generally of the same order. In other words, the interbranch transitions (with a change of helicity) are just as important as intra-branch transitions so that all possible scattering channels should be treated on equal footing. This reflects the known quantum-mechanical fact that there is no general reasons for amplitudes of some scattering channels would be superior to other ones by scattering on a pointlike potential. As opposed to this, two recently published papers^{35,36} denied the necessity of taking into account the impurity scattering induced interbranch transitions, i.e., the off-diagonal components of the matrix $r_{(\mu\nu)}$. One paper states without any supporting evidence that one may omit the interbranch transitions if $\max(\frac{\omega_D}{\alpha p_F}, \frac{1}{\eta}) \ll 1$ (ω_D is the usual BCS cutoff),³⁵ whereas the subsequent paper³⁶ claims also without any proof and disagree-

ing with the previous one that the omission is allowed if $\kappa^{-1} \sim \frac{T_c}{\alpha p_F} \ll 1$. Yet Eqs. (31) and (32), as well as the final expressions (38) and (39) for the dressed spin-vertex do not confirm those statements. Arguments for the neglect of the interbranch transitions at $\kappa^{-1} \ll 1$ are unconvincing to us; particularly in view of the fact that if the arguments were taken seriously, they actually lead to a value of the spin-relaxation time in the normal state (when $\kappa^{-1} = 0$) that differs from τ_{s0} given by the DP theory.³² Second thing is that the parameter δ that enters Eq. (32) through the density of states of two branches falls out of the final expressions for R . The same is true for the all other kernels—they depend on the spin-orbit constant α only through parameters κ and η .

(ii) By means of Eqs. (33)–(35) and by making use of the *ansatz*

$$\hat{M}'(i\epsilon_l) = \sum_{i=0}^3 \tau_i \times \sigma_3 M_{(i)}, \quad (36)$$

where $M_{(i)}$ are c numbers, Eq. (20) can be transformed to the form

$$M_{(0)} = 1 + \frac{\Gamma}{\tilde{E}_n [(\alpha p_F)^2 + \tilde{E}_n^2]} (\tilde{\Delta}^2 M_{(0)} + i\tilde{\epsilon}_n \tilde{\Delta} M_{(1)}),$$

$$M_{(1)} = -\frac{\Gamma}{\tilde{E}_n [(\alpha p_F)^2 + \tilde{E}_n^2]} (i\tilde{\epsilon}_n \tilde{\Delta} M_{(0)} - \tilde{\epsilon}_n^2 M_{(1)}),$$

$$M_{(2)} = M_{(3)} = 0, \quad (37)$$

where $\Gamma = (2\tau)^{-1}$. The unique solution of Eq. (37) has the form

$$M_{(0)} = 1 + \Lambda'_{(0)}, M_{(1)} = \Lambda'_{(1)},$$

$$\Lambda'_{(0)} = \frac{\Gamma}{D_{\parallel}} A_{\parallel}, \Lambda'_{(1)} = -\frac{\Gamma}{D_{\parallel}} C_{\parallel}, \quad (38)$$

where

$$A_{\parallel} = \frac{\tilde{\Delta}_{(s)}^2}{\tilde{E}_l[(\alpha p_F)^2 + \tilde{E}_l^2]}, \quad B_{\parallel} = -\frac{\tilde{\epsilon}_l^2}{\tilde{E}_{(s)}[(\alpha p_F)^2 + \tilde{E}_l^2]},$$

$$C_{\parallel} = \frac{i\tilde{\epsilon}_l\tilde{\Delta}_{(s)}}{\tilde{E}_l[(\alpha p_F)^2 + \tilde{E}_l^2]}, \quad D_{\parallel} = 1 - \Gamma(A_{\parallel} - B_{\parallel}). \quad (39)$$

The extraction of unity from $M_{(0)}$ corresponds to the extraction of the bar spin-vertex from the total impurity-dressed spin vertex.

One can represent $\chi_{s\parallel}$ as a sum of two terms χ_a and χ_b . In the Feynman's diagram language, χ_a is the contribution of a loop where the both response spin vertex and driving spin vertex are the bare ones (the so-called "empty" loop), while the term χ_b is the sum of the ladder-type diagrams with impurity-lines insertions. So the loop corresponding to χ_b contains $\mu_B\hat{X}^{-1}(\tau_0 \times \sigma_3)\hat{X}$ as the response vertex and $\mu_B\hat{X}^{-1}\hat{\Lambda}'\hat{X}$ as the driving spin vertex. Since the sum $\sum_{\epsilon, \mathbf{p}}$ entering χ_b converges absolutely, the p integration may be performed first. However, an analogous sum entering χ_a formally diverges and therefore the summation must be done in the proper order: ϵ_n first, then \mathbf{p} . It is known that in order to interchange the order of summation, one should before perform a summation by parts over ϵ_n .⁴¹ Following Ref. 41, we get

$$\frac{\chi_{a\parallel}}{\chi_n} = 1 - \pi T \sum_l A_{\parallel}, \quad \frac{\chi_{b\parallel}}{\chi_n} = -\pi T \sum_l \frac{\Gamma}{D_{\parallel}} (A_{\parallel}^2 - C_{\parallel}^2), \quad (40)$$

so that

$$\frac{\chi_{s\parallel}(T)}{\chi_n} = 1 - \pi T \sum_l \frac{A_{\parallel}}{D_{\parallel}}, \quad (41)$$

where $\chi_n = 2\mu_B^2 N(0)$ is the 2D normal-state susceptibility. As a result

$$\frac{\chi_{s\parallel}(T)}{\chi_n} = 1 - \pi T \sum_l \frac{\Delta^2}{E_l^3} \left[1 - \frac{(\alpha p_F)^2}{(\alpha p_F)^2 + E_l(E_l + 1/2\tau)} \right]. \quad (42)$$

For a magnetic field parallel to the plane of the electron motion, the same analysis gives rise to

$$\frac{\chi_{s\perp}(T)}{\chi_n} = 1 - \pi T \sum_l \frac{\Delta^2}{E_l^3} \left[1 - \frac{(\alpha p_F)^2(E_l + 1/2\tau)}{(\alpha p_F)^2(2E_l + 1/2\tau) + 2E_l(E_l + 1/2\tau)^2} \right]. \quad (43)$$

Equations (42) and (43) define the spin susceptibility tensor at any impurity concentration and temperature. At $T=0$, the summation over l goes into an integration over ϵ according to the rule $\epsilon_l \rightarrow \epsilon, T \sum_l \rightarrow \int_{-\infty}^{\infty} d\epsilon / 2\pi$. In general, the integrals must be done numerically. In the clean limit $\Delta_{(s)}(0)\tau \gg 1$, Eqs. (42) and (43) give known results.^{20,21} In the dirty limit $\Delta_{(s)}(0)\tau \ll 1$, these equations give rise to

$$\frac{1}{\chi_n}(\chi_{s\parallel}, \chi_{s\perp}) = \begin{cases} \frac{\eta^2 \pi}{8\Delta_{(s)}(0)\tau} \left(1, \frac{1}{2}\right), & \eta^2 \ll \Delta_{(s)}(0)\tau \ll 1 \\ (1, 1), & \Delta_{(s)}(0)\tau \ll \eta^2 \end{cases} \quad (44)$$

at $\eta \ll 1$ and

$$(\chi_{s\parallel}, \chi_{s\perp}) = \chi_n(1, 1) \quad (45)$$

at $\eta \geq 1$. The existence of two intervals of the parameter $\Delta_{(s)}\tau$ in (44) with a different behavior of the spin susceptibility reflects a remarkable feature of the DP mechanism of the spin relaxation. Although the spin-flip transitions are induced by impurity scattering, the very frequent scattering such that the mean-free time τ is shorter than the inverse frequency of the spin precession about the fictitious magnetic field $B_f \sim \alpha p_F$, i.e., $\eta \ll 1$, makes spin-flip transitions less frequent than non-spin-flip scattering $\frac{1}{\tau_{so}} \sim \eta^2 \frac{1}{\tau} \ll \frac{1}{\tau}$.^{32,33} Equation (43) shows that the spin fluctuations in a parity-odd SC are sensitive just to the spin-relaxation time rather than to the mean-free time. The tensor χ_s^{ij} is strongly anisotropic at $\Delta_{(s)}\tau_{so} \gg 1$ but is isotropic at $\Delta_{(s)}\tau_{so} \ll 1$. At $\eta \geq 1$, when τ_{so} and τ are of the same order of magnitude, the two intervals coalesce into one in Eq. (45).

We turn now to the 3D case. All calculations can be carried out along the lines of the 2D case. Just as in the 2D case, the energy spectrum of H_0 consists of two branches but with anisotropic energies

$$\epsilon_{\pm}^{(3D)} = \frac{p^2}{2m} \pm \alpha p z, \quad (46)$$

where z is the sin of the angle between \mathbf{p} and the direction of the polar axis \mathbf{c} . Since

$$\alpha(\mathbf{p} \times \mathbf{c}) \cdot \boldsymbol{\sigma} = \alpha(\mathbf{p}_{\perp} \times \mathbf{c}) \cdot \boldsymbol{\sigma} = \alpha p z (\hat{\mathbf{p}}_{\perp} \times \mathbf{c}) \cdot \boldsymbol{\sigma}, \quad (47)$$

where $\mathbf{p}_{\perp} = \mathbf{p} - \mathbf{c}(\mathbf{c} \cdot \mathbf{p})$ is the component of \mathbf{p} perpendicular to \mathbf{c} , the Green's matrix has the same form as given by Eqs. (6)–(9) but with substitutions

$$\xi_{(\mu)}^{(3D)} = \xi_{(\mu)}(\alpha \rightarrow \alpha z), \quad (48)$$

for $\xi_{(\mu)}$ and

$$\Pi_{(3D)}^{(\pm)}(\mathbf{p}) = \Pi^{(\pm)}(\mathbf{p}_{\perp}), \quad (49)$$

for $\Pi^{(\mu)}(\mathbf{p})$. The equation for the impurity-dressed spin-vertex function has the same form as Eq. (19), where $\sum_{\mathbf{p}}$ now means the 3D integration over the momentum space. It can also be resolved by means of the *ansatz* (36) after 3D counterparts of the kernels P, Q, Y, R, \dots of Eq. (23) are determined. Let us consider as an example the kernel $R_{(3D)}$ —the 3D analog of the kernel R . We have

$$\begin{aligned}
R_{(3D)} &= \frac{1}{\pi N_{(3D)}(0)} \sum_{\mu\nu=\pm} \int \frac{d^3p}{(2\pi)^3} F_{(\mu)}(i\epsilon, p, z) F_{(\nu)}(i\epsilon, p, z) \Pi_{(3D)}^{(\mu)}(\mathbf{p}) \times \Pi_{(3D)}^{(\nu)}(\mathbf{p}) \\
&= \sum_{\mu\nu} \int \frac{p^2 dp}{mp_F \pi} \int_0^\pi \frac{1}{2} \sin \theta d\theta F_{(\mu)}(i\epsilon, p, z) F_{(\nu)}(i\epsilon, p, z) \int \frac{d\phi}{2\pi} \Pi_{(3D)}^{(\mu)}(\mathbf{p}_\perp) \times \Pi_{(3D)}^{(\nu)}(\mathbf{p}_\perp),
\end{aligned} \quad (50)$$

where $N_{(3D)}(0) = mp_F/2\pi^2$ is the 3D density of states on the Fermi level. The last integration in Eq. (50) over the azimuthal angle ϕ does not differ from the analogous integration in the 2D case and hence yields the matrix $R_{(\mu\nu)}$ defined by Eq. (31). So

$$R_{(3D)} = \sum_{\mu\nu} R_{(\mu\nu)} \frac{1}{2} \int_0^\pi \sin \theta d\theta \int \frac{d\xi(p)}{\pi} \left(\frac{p}{p_F} \right) F_{(\mu)}(i\epsilon, p, z) F_{(\nu)}(i\epsilon, p, z), \quad (51)$$

where the factor $\frac{p}{p_F}$ occurs because of the dependence of the 3D density of states on p , $N_{(3D)}(p) = mp/2\pi^2 = N_{(3D)}(0) \frac{p}{p_F}$. The p integration in Eq. (51) can be performed with the help of the theory of residues; as a result we have an expression which differs from its 2D counterpart, given by the right-hand side of the Eq. (32), by only the changes $\alpha \rightarrow \alpha z$ and $\delta \rightarrow 2\delta z$. Finally, we obtain

$$R_{(3D)} = \left[\frac{\tilde{\Delta}_{(s)}^2}{4\tilde{E}_n} \right] \frac{1}{2} \int_{-1}^1 dz w_{3D}(z), \quad (52)$$

where

$$w_{3D}(z) = w(\alpha \rightarrow \alpha z), \quad (53)$$

with w given by Eq. (34). Thus, $R_{(3D)}$ is obtained from its 2D counterpart R by the substitution

$$w \rightarrow \langle w \rangle = \frac{1}{2} \int_{-1}^1 w_{3D}(z) dz. \quad (54)$$

The same is true with respect to other kernels $P_{(3D)}, Q_{(3D)}, \dots$. The substitution (54) is equivalent to the substitution

$$\frac{1}{(\alpha p_F)^2 + \tilde{E}_n^2} \rightarrow \frac{1/2}{\alpha p_F \sqrt{(\alpha p_F)^2 + \tilde{E}_n^2}} \ln \frac{\sqrt{(\alpha p_F)^2 + \tilde{E}_n^2} + \alpha p_F}{\sqrt{(\alpha p_F)^2 + \tilde{E}_n^2} - \alpha p_F}, \quad (55)$$

in Eq. (34). With the aid of these results and following along the lines of the 2D case, we get

$$\frac{\chi_{s\parallel}^{(3D)}}{\chi_n^{(3D)}} = 1 - \pi T \sum_{\epsilon_l} \left(\frac{\Delta_{(s)}}{E_l} \right)^2 \frac{K(\epsilon_l)}{\tilde{E}_l - \Gamma K(\epsilon_l)} \quad (56)$$

and

$$\frac{\chi_{s\perp}^{(3D)}}{\chi_n^{(3D)}} = 1 - \pi T \sum_{\epsilon_l} \left(\frac{\Delta_{(s)}}{E_l} \right)^2 \frac{1 + K(\epsilon_l)}{2\tilde{E}_l - \Gamma[1 + K(\epsilon_l)]}, \quad (57)$$

where

$$K(\epsilon_l) = \frac{\tilde{E}_l^2/2}{\alpha p_F \sqrt{(\alpha p_F)^2 + \tilde{E}_l^2}} \ln \frac{\sqrt{(\alpha p_F)^2 + \tilde{E}_l^2} + \alpha p_F}{\sqrt{(\alpha p_F)^2 + \tilde{E}_l^2} - \alpha p_F}. \quad (58)$$

At $T=0$ and in the clean limit $\Delta_{(s)}\tau \gg 1$, Eqs. (56) and (57) yield

$$\frac{1}{\chi_n^{(3D)}} (\chi_{s\parallel}^{(3D)}, \chi_{s\perp}^{(3D)}) = \left(1, \frac{1}{2} \right) \times \begin{cases} \left(\frac{2\alpha p_F}{3\Delta_{(s)}(0)} \right)^2, & \alpha p_F \ll \Delta_{(s)}(0) \\ 1, & \alpha p_F \gg \Delta_{(s)}(0) \end{cases}. \quad (59)$$

In the dirty limit $\Delta_{(s)}(0)\tau \ll 1$, these give rise to

$$\frac{1}{\chi_n^{(3D)}} (\chi_{s\parallel}^{(3D)}, \chi_{s\perp}^{(3D)}) = \begin{cases} \frac{\eta^2 \pi}{12\Delta_{(s)}(0)\tau} \left(1, \frac{1}{2} \right), & \eta^2 \ll \Delta_{(s)}(0)\tau \ll 1 \\ (1, 1), & \Delta_{(s)}(0)\tau \ll \eta^2 \end{cases} \quad (60)$$

at $\eta \ll 1$ and

$$(\chi_{s\parallel}^{(3D)}, \chi_{s\perp}^{(3D)}) = \chi_n^{(3D)}(1, 1) \quad (61)$$

at $\eta \geq 1$. Thus, the results for the 3D case are completely analogous to those for the 2D case.

III. CONCLUDING REMARKS AND SUMMARY

Equations (44) and (45) as well as Eqs. (60) and (61) describe a remarkable fact: a SC with the s -wave singlet pairing and hence with a *fully gapped* energy spectrum can show the spin susceptibility of the normal, gapless state because of the BSOC. In other words, the SC can have familiar thermal properties, for example, an activation-type temperature dependence of the specific heat $\sim \exp[-\Delta_{(s)}(0)/T]$, but at the same time it can show the temperature-independent spin susceptibility as if it has *nodes* in the energy gap. It should be noted that scattering on impurity atoms with a large atomic number $Z \gg 1$ is also able to result in an analogous effect in a conventional SC. But because the ratio b/a of the spin-orbital part of the scattering amplitude, $u(\mathbf{p}, \mathbf{p}') = a(\mathbf{p} - \mathbf{p}') + ib(\mathbf{p} - \mathbf{p}') p_F^{-2} (\mathbf{p} \times \mathbf{p}') \cdot \sigma$, to the scalar one is of order $Ze^2/\hbar c$ this could be under a severe condition τT_c

$\ll \frac{n_Z}{n} (Z \frac{e^2}{\hbar c})^2$,¹² where n_Z is the concentration of the heavy atoms and n_{imp} is the total impurity concentration.

Thus, there is a variety of scenarios of temperature independent χ_s .^{25,26,28,29} One should also keep in mind that an applied magnetic field usually used in experiments on the nuclear-magnetic resonance is comparable with the critical field of a sample. It is known that in the case of conventional SCs with the ISOC, the effect of the field is to increase $\chi_s(0)$.⁴⁴ A corresponding theory for SCs with the BSOC is not developed yet, however, it seems that an analogous effect should be present in such SCs too. Therefore, the fact that $\chi_s(0) \approx \chi_n(T_c)$ alone does not allow one to judge definitely the character of the pairing and the energy spectrum of a given parity-odd SC. It is necessary to have more spin-dependent characteristics calculated for both “clean” and “dirty” cases, for example, the upper critical magnetic field $H_{c2}(T)$ as a function of the angle between the field and the polar axes and the magnetic-field dependence of the critical current $J_c(H)$, to compare them to experimental data. The finite value of $\chi_s(0)$ under the condition of a finite-energy gap apparently means the ability of the Cooper-pair condensate to participate in the paramagnetic magnetization under the action of an external magnetic field. Therefore, a study of the decay of the magnetization after the inducing magnetic field is switched off could shed light on the nature of the magnetization and help to understand the gap structure. Correspondingly, it would be interesting to calculate the dynamic spin susceptibility $\chi_s(\omega, T)$ for all scenarios mentioned.

The results obtained are applicable, in the first instance, to SCs without strong magnetic correlations. Perhaps the most simple of those are quasi-2D asymmetric SCs: (a) There are experimental indications in favor of the existence of superconductive states localized on a surface of nonsuperconductive crystals.¹⁸ In this case, the intracrystalline electric field and hence the BSOC is caused by the crystal lattice disturbance near the surface. (b) Owing to the proximity effect, the parity-odd superconductivity can be induced in a polar non-superconductive metal near the contact with a conventional SC. (c) Ultrathin films of a conventional SC, for example, Al or Be, covered with a monolayer of another metal, for example, Au, could also serve as a model system of a 2D asymmetric SC.⁴⁵ In the latter case, spin-orbit coupling can be due to both a double electric layer formed on the junction of two conductors with different work functions¹⁷ and the strong spin-orbit component of the electron scattering on Au ions.

The situation with bulk heavy fermion SCs of polar symmetry, in particular, with CePt₃Si on which the major part of experiments published so far was performed, seems much more complicated. Almost total and simultaneous destruction of antiferromagnetism and superconductivity by substitution of 2% La for Ce (Ref. 46) indicates that the Kondo-like exchange interaction is more important for superconductivity in this compound than the broken central symmetry because the BSOC is apparently of the same order in both CePt₃Si and LaPt₃Si. Therefore any approach that ignores the interaction responsible for the Kondo-lattice formation, like that used in this paper, has a restrictive applicability and may be used, in the first instance, for an analysis of general, quality aspects of the superconductive state of this material.

The next problem is whether this superconductor is clean or dirty, i.e., the question about the mean-free time τ . The border between clean and dirty superconductivity lies at $\tau_b = (\pi T_c)^{-1}$. So $T_c = 0.65$ K (Ref. 46) corresponds to $\tau_b \approx 0.4 \times 10^{-11}$ s. This time does not seem too short if one takes into account the many-element character of the compound; for comparison, the electron free time in single-crystal high-purity potassium $\tau = 1.6 \times 10^{-10}$ s is only one and a half orders of magnitude longer.⁴⁷ Reference²⁴ put CePt₃Si to clean SCs, that is equivalent to the admission that $\tau > \tau_b$. One of arguments for that was the measured value of the rate of a change of the upper critical field $H'_{c2}(\text{meas}) = 8.5$ T/K ($H'_{c2} \equiv dH_{c2}/dT$) was found to be much greater than the calculated one, $H'_{c2}(\text{calc}) = 0.77$ T/K. It is unclear, however, which theory was used; to the best of our knowledge, correct calculations of $H_{c2}(T)$ for parity-odd SCs was never published yet. Likewise, the relation $H_{c2}(0) = \frac{\Phi_0}{2\pi\xi_0^2}$ between the coherence length $\xi_0 = \frac{v_F}{2\pi T_c}$ and $H_{c2}(0)$ used in Ref. 24 is hardly valid in SCs with the BSOC too. An estimate for the Fermi velocity obtained was $v_F \approx 5 \times 10^5$ cm/s. The value of v_F about velocity of sound is unusual and requires additional evidences. Thus, it does not appear that the classifying CePt₃Si as a clean SC can be regarded as definitive; it would seem that further work is needed to resolve the problem. Besides, τ might be inferred from experiments performed on the normal phase, for example, from the form of the IR absorption line near the cyclotron resonance as a function of the applied frequency; therefore, the value of τT_c could be obtained without reference to superconducting properties.

However, the central question is that about the value of the spin-orbit constant α . Just as for any material constant, it is most reliable to extract α from direct experimental measurements without relying on any theoretical estimates. In the case of asymmetric 2D systems mentioned above and single crystals of polar symmetry (or polycrystalline samples with a primary direction of the polar axis), a candidate experiment could be detecting a change in the *value* of the critical current arising with the switching of the (relatively weak) applied magnetic field. This asymmetric effect is a consequence of the presence of a term bilinear in the supercurrent \mathbf{J}_s and the magnetic field of the form $(\mathbf{c} \times \mathbf{H}) \cdot \mathbf{J}_s$ (where \mathbf{c} is the polar axis) in the free energy of a polar SC. This is a truly symmetry effect and hence should not be very sensitive to the magnetic ordering and other particular properties of the SC. Its magnitude is strictly proportional to δ .⁴³

If, because of features of the electronic band structure of a compound, δ appears to be not too small, an evaluation of χ_s in an impure SC becomes a much more involved problem. In principle, the triplet channel of pairing can be included in the formalism presented (see, the Appendix B and Refs. 28 and 29). However, $\Delta_{(i)}$ is not a unique source of an additional δ dependence. Another sources are a finite effective range of the real impurity potential and an exact form of the irreducible two-particle amplitude in the Cooper channel. The point is that due to the BSOC, the exact T matrix of scattering on a *scalar* impurity of a *finite* radius acquires terms of the order of δ with a nontrivial spin-momentum dependence. An attempt to determine such a dependence for the 2D semiconductor systems has been undertaken recently⁴⁸ (an analogous

term in the Cooper channel T matrix was pointed out too^{28,29}). Under the circumstances, the lifetime of an electron becomes a function of its helicity $\tau_{(+)}^{-1} - \tau_{(-)}^{-1} \sim \delta$, the vertex function \hat{M}'_j of Eq. (19) depends on both ϵ_j and \mathbf{p} , and the finding of a solution of Eq. (19) becomes an intensive task. Besides, the phase of the Cooper pair condensate induced by an applied magnetic field,²⁰ whose contribution to the susceptibility can be shown to be negligibly small at $\delta \ll 1$, should be taken into consideration if $\delta \sim 1$.

In summary, we have shown that the effect of the DP mechanism of the spin-flip impurity scattering is to increase the value of the spin susceptibility of a parity-odd SC up to that of the normal state. The main consequence of this result is that a weak temperature dependence of the spin susceptibility of parity-odd SCs below T_c or even the total absence of such a dependence is not necessary due to the presence of the nodes in the energy gap but can well be a result of a disorder. In other words, the impure polar SC can behave as a gapless SC with respect to the Zeeman interaction with an applied magnetic field remaining in a full-gap state. An analogous result may appear to be valid also for parity-odd SCs of cubic symmetry, where the BSOC of the Dresselhaus-type can be essential. The methods of treating the impurity scattering described in the present paper will be useful in calculations of other equilibrium characteristics and necessary for an analysis of nonequilibrium processes in impure parity-odd SCs.

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APPENDIX A

The purpose of this appendix is to support the statement that the superconducting correlations in the model defined by Eqs. (2)–(5) do contain the triplet part in spite of the singlet form of the order parameter $\Delta_{\alpha\beta}^{(s)} = \Delta_{(s)} g_{\alpha\beta}$.

To see how this happens, it is sufficient to consider the anomalous Green function $F(i\epsilon, \mathbf{p})_{\alpha\beta}$ of a clean SC near T_c , when this function has the simplest form

$$F(i\epsilon, \mathbf{p})_{\alpha\beta} \simeq G_{\alpha\beta}^{(0)}(i\epsilon, \mathbf{p}) \Delta_{\rho\kappa} (-1) G_{\kappa\beta}^{(0)\dagger}(-i\epsilon, -\mathbf{p}). \quad (\text{A1})$$

Here, $G_{\alpha\beta}^{(0)}(i\epsilon, \mathbf{p}) = \sum_{\mu=\pm} \Pi_{\alpha\beta}^{(\mu)}(\mathbf{p})$, $G_{(\mu)}^{(0)}(i\epsilon, p) G_{(\mu)}^{(0)\dagger}(i\epsilon, p) = [i\epsilon - \xi_{(\mu)}(p)]^{-1}$ is the electron Green function of the normal state. By making use of the equalities

$$g \cdot \Pi^{(\mu)\dagger}(-\mathbf{p}) \cdot g^t = \Pi^{(\mu)}(\mathbf{p}), \quad \Pi^{(\mu)}(\mathbf{p}) \cdot \Pi^{(\nu)}(\mathbf{p}) = \delta^{\mu\nu} \Pi^{(\mu)}(\mathbf{p}), \quad (\text{A2})$$

one can represent the F function as the sum of the singlet component

$$\Delta_{(s)}(T) g_{\alpha\beta} \left[\sum_{\mu} \frac{1}{2} G_{(\mu)}^{(0)}(i\epsilon, p) (-1) G_{(\mu)}^{(0)\dagger}(-i\epsilon, p) \right] \quad (\text{A3})$$

and the triplet component

$$\begin{aligned} & \Delta_{(s)}(T) A_{ij\hat{p}i}(\sigma_j g)_{\alpha\beta} \\ & \times \left[\sum_{\mu} \frac{1}{2} \text{sgn}_{\mu} G_{(\mu)}^{(0)}(i\epsilon, p) (-1) G_{(\mu)}^{(0)\dagger}(-i\epsilon, p) \right], \end{aligned} \quad (\text{A4})$$

where $A_{ij} = e_{ijk} c_k$. As it is seen, the source of this triplet component is the broken mirror symmetry of the *normal* state rather than the triplet channel of the pairing interaction. It is just for this reason the zero-temperature spin susceptibility was found to be finite in Refs. 20–22. The presence of impurities is not able to eliminate this source of the triplet correlations. Moreover, as the present calculations show, the impurity scattering only modifies the influence of this source but not suppress it.

APPENDIX B

In the main body of the text, we used a BCS-like model, where all the channels of the pairing interaction except for the s -wave channel were neglected. The main purpose of this appendix is to sketch out the Green's function formalism which takes into account the pairing in both the s - and p -wave channels and estimate the relative role of the triplet order parameter that appears owing to the p -wave pairing. The corresponding formalism for the clean case is mainly known.^{13,20} An extension to impure systems can be carried out by standard methods⁴¹ along the lines of the main text, and we shall therefore omit some details. For the sake of definiteness, we consider a 2D SC.

If, in addition to the s -wave part of Eq. (5), one includes the p -wave part

$$\begin{aligned} V_p^{\alpha\beta|\gamma\delta}(\mathbf{r}_1, \mathbf{r}_2 | \mathbf{r}_3, \mathbf{r}_4) &= \lambda_p (\sigma^k g)_{\alpha\beta} (g \sigma^k)_{\gamma\delta} \delta(\mathbf{r}_1 - \mathbf{r}_2) \\ & \times \delta(\mathbf{r}_3 - \mathbf{r}_4) \delta(\mathbf{r}_1 - \mathbf{r}_3) \nabla_{12}^i \nabla_{34}^i, \end{aligned} \quad (\text{B1})$$

in the pairing Hamiltonian, where $\nabla_{12}^i = (\partial/\partial r_2^i - \partial/\partial r_1^i)/2ik_F$; equations of the main text undergoes some changes. The self-consistency equation becomes

$$\begin{aligned} \Delta_{\alpha\beta}(\mathbf{p}; T) &= -T \sum_{\epsilon} \int \frac{d^2 p}{2\pi} V_{\alpha\beta|\gamma\rho}(\mathbf{p}, \mathbf{k}) F_{\rho\gamma}(i\epsilon, -\mathbf{k}), \\ V_{\alpha\beta|\gamma\rho}(\mathbf{p}, \mathbf{k}) &= \lambda_s g_{\alpha\beta} g_{\gamma\rho}^t + \lambda_p (\sigma^j g)_{\alpha\beta} (\hat{\mathbf{p}} \cdot \hat{\mathbf{k}}) (g^t \sigma^j)_{\gamma\rho}. \end{aligned} \quad (\text{B2})$$

Just as in the clean case, the spin structure of the order-parameter matrix takes the form

$$\begin{aligned} \Delta_{\alpha\beta}(\mathbf{p}) &= \Delta_{(s)} g_{\alpha\beta} + \Delta_{(t)}(p) (\hat{\mathbf{p}} \times \mathbf{c} \cdot \boldsymbol{\sigma})_{\alpha\gamma} g_{\gamma\beta} \\ &= \sum_{\mu=\pm} \Pi_{\alpha\gamma}^{(\mu)}(\hat{\mathbf{p}}) \Delta_{(\mu)} g_{\gamma\beta}, \quad \Delta_{(\pm)} = \Delta_{(s)} \pm \Delta_{(t)}. \end{aligned} \quad (\text{B3})$$

This means that the energy branched of different helicities acquire different energy gaps. Accordingly, Eq. (9) transform into

$$\begin{pmatrix} G_{(v)}(i\epsilon_n, p) \\ F_{(v)}(i\epsilon_n, p) \end{pmatrix} = \frac{1}{(i\tilde{\epsilon}_n)^2 - \xi_{(v)}^2 - \tilde{\Delta}_{(v)}^2(n)} \begin{pmatrix} i\tilde{\epsilon}_n + \xi_{(v)} \\ \tilde{\Delta}_{(v)} \end{pmatrix}. \quad (\text{B4})$$

Equations (11) and (12) which define the c -number functions $\tilde{\epsilon}_n$ and $\tilde{\Delta}_{(v)}$ remain valid. With this information, we are in a position to estimate $\Delta_{(t)}$. After substituting Eq. (B3), the self-consistency Eq. (B2) transforms into the system of equations

$$\Delta_{(s)} = -\lambda_s T \sum_{\epsilon} \int \frac{kdk}{2\pi} [F_{(+)}(i\epsilon, k) + F_{(-)}(i\epsilon, k)], \quad (\text{B5})$$

$$\Delta_{(t)} = \frac{\lambda_p T}{2} \sum_{\epsilon} \int \frac{kdk}{2\pi} [F_{(+)}(i\epsilon, k) - F_{(-)}(i\epsilon, k)]. \quad (\text{B6})$$

Actually the sum and integral in Eqs. (B5) and (B6) involve a logarithmic divergence for large energies. This divergence is known to be removed by a cutoff procedure. BCS applied the cutoff in the kinetic-energy variable $\xi(k)$.¹ As it was first noticed in Ref. 49, in the case of an impure SC, it is convenient instead to cut off the frequency ϵ_n . We shall also use this procedure. The system of Eqs. (B5), (B6), (11), and (12) can be solved by means of the iteration method. Let us assume that $\Delta_{(t)} \sim \delta \Delta_{(s)} \ll \Delta_{(s)}$. Then one can drop $\Delta_{(t)}$ in the right-hand side of Eq. (B5) that reduces Eq. (B5) to the form

$$\Delta_{(s)} = -\lambda_s T \sum_{\epsilon} \int \frac{kdk}{2\pi} \sum_{\nu=\pm} \frac{\tilde{\Delta}_{(s)}}{(i\tilde{\epsilon})^2 - \xi_{(\nu)}^2 - \tilde{\Delta}_{(s)}^2}. \quad (\text{B7})$$

Further let us assume that at $\delta \ll 1$,

$$\frac{\tilde{\epsilon}_n}{\epsilon_n} = \frac{\tilde{\Delta}_{(s)}}{\Delta_{(s)}}, \quad (\text{B8})$$

and note that to an accuracy of δ^2 ,

$$\frac{kdk}{2\pi} \cong N(0) d\xi_{(\pm)} (1 \mp \delta). \quad (\text{B9})$$

Then it is seen that the k integration reduces Eq. (B7) to the standard BCS self-consistency equation

$$\Delta_{(s)} = N(0) \pi T \sum_{|\epsilon_n| \leq \omega_D} \frac{\Delta_{(s)}}{\sqrt{\epsilon_n^2 + \Delta_{(s)}^2}}. \quad (\text{B10})$$

Thus $\Delta_{(s)} \cong \Delta_{\text{BCS}}$. Let, for the sake of simplicity, $\lambda_p/\lambda_s \ll 1$. Then one can drop $\Delta_{(t)}$ in the functions $F_{(\pm)}(i\epsilon, k)$ entering the right-hand side of Eq. (B6) too. After that Eq. (B6) becomes

$$\Delta_{(t)} = \frac{\lambda_p T}{2} \sum_{\epsilon} \int \frac{kdk}{4\pi} \sum_{\nu=\pm} \text{sign}(\nu) \frac{\tilde{\Delta}_{(s)}}{(i\tilde{\epsilon})^2 - \xi_{(\nu)}^2 - \tilde{\Delta}_{(s)}^2}. \quad (\text{B11})$$

By making use of Eqs. (B8) and (B9), the integration over k transforms the right-hand side of Eq. (B11) to

$$\lambda_p \delta N(0) \pi T \sum_{|\epsilon_n| \leq \omega_D} \frac{\Delta_{(s)}}{\sqrt{\epsilon_n^2 + \Delta_{(s)}^2}}. \quad (\text{B12})$$

Hence,

$$\frac{\Delta_{(t)}}{\Delta_{(s)}} \cong \delta \left(\frac{\lambda_p}{2\lambda_s} \right), \quad (\text{B13})$$

just as in the clean case.²⁰ It remains to check the consistence of the assumption (B8). Equations (11) and (12) together with Eq. (B4) give

$$i\tilde{\epsilon}(\epsilon_n) = i\epsilon - \frac{1}{2} \int \frac{d\xi(k)}{2\pi\tau} \sum_{\nu=\pm} \frac{i\tilde{\epsilon} + \xi_{(\nu)}}{(i\tilde{\epsilon})^2 - \xi_{(\nu)}^2 - |\tilde{\Delta}_{(v)}|^2}, \quad (\text{B14})$$

$$\tilde{\Delta}_{(\mu)}(\epsilon_n) = \Delta_{(\mu)} - \frac{1}{2} \int \frac{d\xi(k)}{2\pi\tau} \sum_{\nu=\pm} \frac{\tilde{\Delta}_{(v)}}{(i\tilde{\epsilon})^2 - \xi_{(\nu)}^2 - |\tilde{\Delta}_{(v)}|^2}. \quad (\text{B15})$$

By making use of Eq. (B9) and omitting δ^2 terms, one can reduce Eq. (B14) to

$$i\tilde{\epsilon} = i\epsilon - \frac{1}{2} \int \frac{d\xi(k)}{2\pi\tau} \sum_{\nu=\pm} \frac{i\tilde{\epsilon}}{(i\tilde{\epsilon})^2 - \xi_{(\nu)}^2 - |\tilde{\Delta}_{(v)}|^2}, \quad (\text{B16})$$

that to the same accuracy is equivalent to

$$\frac{\tilde{\epsilon}}{\epsilon} = 1 + \frac{\tilde{\epsilon}}{2\epsilon} \left[\frac{1-\delta}{\sqrt{\epsilon^2 + \tilde{\Delta}_{(+)}^2}} + \frac{1+\delta}{\sqrt{\epsilon^2 + \tilde{\Delta}_{(-)}^2}} \right]. \quad (\text{B17})$$

The right-hand side of Eq. (B17) is an even function of δ and hence dropping δ^2 corrections can be taken at $\delta=0$. Then Eq. (B17) coincides with an analogous equation for an impure conventional SC.⁶ Thus we have

$$\frac{\tilde{\epsilon}_n}{\epsilon_n} = u(\epsilon_n), u(\epsilon_n) = 1 + \frac{1/2\tau}{\sqrt{\epsilon_n^2 + \Delta_{(s)}^2}}. \quad (\text{B18})$$

Consider now Eq. (B15). Since the second term of right-hand side of Eq. (B15) is the same for $\mu=\pm$, we have

$$\tilde{\Delta}_{(t)} = \Delta_{(t)}, \quad (\text{B19})$$

i.e., the triplet part of the order parameter is not subject to impurity renormalization at the s -wave impurity scattering assumed. This fact becomes evident if one considers the first impurity-induced correction to $\Delta_{(t)}^{(i)}(\mathbf{p})_{\alpha\beta}$. At $T \approx T_c$ (where the correction has the simplest form), it is

$$\begin{aligned} & \frac{1}{m\tau} \sum_{\mathbf{p}} G_{\alpha\gamma}^{(0)}(i\epsilon, \mathbf{p}) \Delta_{(t)} [(\mathbf{p} \times \mathbf{c}) \cdot \boldsymbol{\sigma}]_{\gamma\rho} g_{\rho\kappa} (-1) G_{\kappa\beta}^{(0)t}(-i\epsilon, -\mathbf{p}) \\ & = \frac{1}{m\tau} g_{\alpha\beta} \sum_{\mathbf{p}} \sum_{\nu=\pm} \frac{\nu}{2} G_{(\nu)}^{(0)}(i\epsilon, p) \Delta_{(t)} (-1) G_{(\nu)}^{(0)}(-i\epsilon, p). \end{aligned} \quad (\text{B20})$$

It is seen, first, that this expression has the singlet spin struc-

ture rather than the triplet one and, second, that its magnitude is of the order of δ^2 (one δ comes from $\Delta_{(t)} \sim \delta$ and other from the small difference between the densities of states on two Fermi surfaces).

An analogous reasoning leads to that to the accuracy of δ^2 corrections

$$\frac{\tilde{\Delta}_{(s)}}{\Delta_{(s)}} = u(\epsilon_n). \quad (\text{B21})$$

Indeed, by substituting Eqs. (B19) and (B20) into Eq. (B15) we see that the integral term on the right-hand side of Eq. (B15) is an even function of δ and therefore it can be taken at $\delta=0$ to the same accuracy, i.e., written in the form

$$\int \frac{d\xi(k)}{2\pi\tau} \frac{\tilde{\Delta}_{(s)}}{(i\tilde{\epsilon})^2 - \xi^2 - |\tilde{\Delta}_{(s)}|^2}. \quad (\text{B22})$$

Then Eq. (B15) can be written as

$$\tilde{\Delta}_{(s)} \pm \tilde{\Delta}_{(t)} = \Delta_{(s)} \pm \Delta_{(t)} - \int \frac{d\xi(k)}{2\pi\tau} \frac{\tilde{\Delta}_{(s)}}{(i\tilde{\epsilon})^2 - \xi^2 - |\tilde{\Delta}_{(s)}|^2}. \quad (\text{B23})$$

Upon using Eq. (B19), Eq. (B23) reduces to the standard equation for $\Delta_{(s)}$.⁶ This proves Eq. (B21).

Thus, Eqs. (B18)–(B21) provide the self-consistence solution for Eqs. (11), (12), and (B2) at $\delta \ll 1$. Further, $\Delta_{(t)}$ appears in the Green's functions through combinations

$$\tilde{\Delta}_{\pm}(\epsilon_n) = \Delta_{(s)}u(\epsilon_n) \pm \Delta_{(t)}. \quad (\text{B24})$$

In the dirty limit $T_c\tau \ll 1$, when $u(\epsilon_n) \sim (2\tau\Delta_{(s)})^{-1}$, the ratio of the second term in Eq. (B24) to the first one is of the order of $(T_c\tau)\delta$, i.e., is even smaller than in the clean limit. It should be stressed that although the neglect of $\Delta_{(t)}$ is apparently correct for calculations of equilibrium properties of polar SCs with $\delta \ll 1$, the account of $\Delta_{(t)}$ can appear to be necessary in some subtle electrodynamic problems where the difference between the energy gaps of electrons with opposite helicities may play an essential role.

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